The results presented indicate that under stationary conditions the second-order moment $\langle f_1^2 \rangle$ can vary even for constant f_0 over an extremely wide range, depending on the relations between the parameters α, β^2, ε, ω^2, $\varkappa$, L.

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NON-STATIONARY FRICTIONAL HEATING IN SLIDING COMPRESSIBLE ELASTIC BODIES†

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The redistribution of contact pressure due to the influence of the thermal energy generated by the friction between two sliding elastic isotropic bodies is investigated. The plastic strength of the friction pair can be represented as the sum of the force and temperature components of the stress tensor. A method for controlling plastic deformations connected with wear is proposed.

1. WE CONSIDER the problem of contact between two elastic heterogeneous bodies, one of which is a half-space, while the other one is bounded by an axially-symmetric surface of circular shape. The bodies are in contact under the action of a compressive force P and a shear force fP, f being the coefficient of friction. The surface of the half-space is sliding at a constant speed V on the stationary axially symmetric surface (an irregularity) in the direction of the x axis. As a result of friction, heat is generated within the area of contact, which gives rise to the heat flux

$$Q(r) = \gamma f V p(r), \quad r \leqslant a \tag{1.1}$$

into the stationary body. Here γ is distribution coefficient of the heat flux, $p(r)$ is the contact pressure in the corresponding isothermal contact problem [1]

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$$p(r) = p_0(1-\rho^2)^{-1/2} \sum_{m=0}^{\infty} X_m P_m^*(\rho), \quad \rho = r/a$$

$$X_m = \frac{2E^* G_m \sigma_m}{a^3 p_0 \lambda_m^2}, \quad G_m = \int_0^a \frac{rG(r) P_m^*(\rho)}{\sqrt{1-\rho^2}} dr \tag{1.2}$$

$$\lambda_m = \sqrt{\frac{\pi}{2}} \frac{(2m-1)!!}{2^m m!}, \quad P_m^*(\rho) = P_{2m}(\sqrt{1-\rho^2}), \quad r^2 = x^2 + y^2$$

$$G(r) = \Delta - g(r), \frac{1}{2E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}, \quad p_0 = \frac{P}{\pi a^2}, \quad \sigma_m = \frac{1}{2} + 2m,$$
$$m = 0, 1, \ldots$$

$z = g(r)$ is the equation of the surface of the irregularity, Δ is the vertical displacement of the centre of mass of the irregularity, E_i and ν_i are Young's moduli and Poisson's ratio of the material of the stationary body ($i = 1$), and the half-space ($i = 2$), and $P_m(\cdot)$ are Legendre polynomials. From the condition of equilibrium

$$2\pi \int_0^a rp(r)\, dr = P$$

it follows that $X_0 = 1/2$. A fairly complete list of existing forms of real contact surfaces is given in [2].

To investigate the thermal regime of the axially symmetric body, we have to find the solution of the heat conduction equation

$$\nabla^2 T = \frac{a^2}{k} \frac{\partial T}{\partial t}, \quad \nabla^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial Z^2} \tag{1.3}$$

(T denotes the temperatures, t is the time, k is the thermal diffusivity of the material of the irregularity and $Z = z/a$) with the boundary conditions

$$\frac{\partial T}{\partial Z} \begin{cases} -\Lambda(1-\rho^2)^{-1/2} \sum_{m=0}^{\infty} X_m P_m^*(\rho), & 0 \leqslant \rho < 1, \quad Z = 0 \\ 0, \quad \rho > 1, \quad Z = 0 & (\Lambda = Q_0 a/\lambda, \ Q_0 = \gamma f V p_0) \end{cases} \tag{1.4}$$

$$\frac{\partial T}{\partial \rho} = 0, \quad \rho = 0, \quad 0 \leqslant Z < \infty \tag{1.5}$$

the regularity conditions

$$T \to 0 \quad \text{as} \quad \rho \to \infty, \quad T \to 0 \quad \text{as} \quad Z \to \infty \tag{1.6}$$

and the initial conditions

$$T = 0 \ \text{for} \ t = 0 \tag{1.7}$$

(λ is the thermal conductivity of the material of the stationary body).

Applying to (1.3) a Hankel integral transformation order zero with respect to ρ and then a Laplace transformation with respect to t and using the conditions (1.5)–(1.7), we obtain

$$\frac{\partial^2 \bar{\bar{T}}}{\partial Z^2} - \xi^2 \bar{\bar{T}} = \frac{a^2 s}{k} \bar{\bar{T}} \tag{1.8}$$

$$\bar{\bar{T}}(\xi, Z, s) \int_0^{\infty} e^{-st} \bar{T}(\xi, Z, t)\, dt$$

$$\bar{T}(\xi, Z, t) = \int_0^{\infty} \rho J_0(\xi\rho) T(\rho, Z, t)\, d\rho$$

[$J_0(\cdot)$ is a Bessel function]. The transformed conditions (1.4) and (1.6) yield

$$\partial \bar{\bar{T}}/\partial Z = -\Lambda\varphi(\xi)/s \ \text{for} \ Z = 0, \ \bar{\bar{T}} \to 0 \quad \text{as} \quad Z \to \infty \tag{1.9}$$

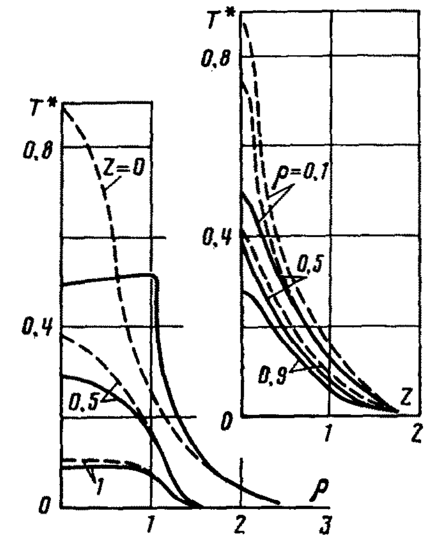

FIG. 1.

The solution of Eq. (1.8) that satisfies (1.9) has the form

$$\bar{\bar{T}}(\xi, Z, s) = \Lambda\varphi(\xi)[s\sqrt{\xi^2 + a^2 s/k}\exp(Z\sqrt{\xi^2 + a^2 s/k})]^{-1} \tag{1.10}$$

Applying inverse Laplace and Hankel transformations to (1.10), we get ($\mathrm{Fo} = a^{-2}kt$ is the Fourier criterion)

$$T(\rho, Z, t) = \Lambda\int_0^\infty \varphi(\xi)\,\Phi_0(\xi, Z, \mathrm{Fo})\,J_0(\xi\rho)\,d\xi \tag{1.11}$$

$$\Phi_0(\xi, Z, \mathrm{Fo}) = \frac{1}{2}\left[e^{-\xi Z}\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} - \xi\sqrt{\mathrm{Fo}}\right) - e^{\xi Z}\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} + \xi\sqrt{\mathrm{Fo}}\right)\right]$$

The integral on the right-hand side of (1.11) can be estimated numerically using the DQAGS procedure of the QUADPACK package [3]. For $\mathrm{Fo} = 1$ and $\nu_1 = 0.3$, the values of the dimensionless temperature $T^* = T/\Lambda$ for a cylindrical irregularity ($m = 0$; the solid line) and a spherical irregularity ($m = 1$; the dashed line) are shown in Fig. 1.

2. The non-uniform temperature distribution (1.11) gives rise to thermal stress inside the body. If there are no mass forces the elastic displacements induced by the temperature field in an elastic body can be determined from the equation

$$u_{i,jj} + \frac{1}{1-2\nu_1}u_{j,ji} = \frac{2(1+\nu_1)}{1-2\nu_1}\alpha_t T_{,i} \tag{2.1}$$

Here α_t is the coefficient of linear thermal expansion of the material of the axially symmetric body. Introducing the thermoelastic potential $\Phi_{,i} = u_i$, we can write Eq. (2.1) in terms of the dimensionless coordinates ρ and Z as the following equivalent differential equation:

$$\nabla^2\Phi = \beta T, \quad \beta = a^2\alpha_t(1+\nu_1)/(1-\nu_1) \tag{2.2}$$

We can determine the temperature stress from the formulas

$$\sigma_{r1} = \frac{2\mu}{a^2}\left(\frac{\partial^2\Phi}{\partial\rho^2} - \nabla^2\Phi\right), \quad \sigma_{\theta 1} = \frac{2\mu}{a^2}\left(\frac{1}{\rho}\frac{\partial\Phi}{\partial\rho} - \nabla^2\Phi\right)$$

$$\sigma_{z1} = \frac{2\mu}{a^2}\left(\frac{\partial^2\Phi}{\partial Z^2} - \nabla^2\Phi\right), \quad \sigma_{rz1} = \frac{2\mu}{a^2}\frac{\partial^2\Phi}{\partial\rho\partial Z}, \quad \mu = \frac{E_1}{2(1+\nu_1)} \tag{2.3}$$

We shall construct a solution of Eq. (2.2) that satisfies the boundary conditions

$$\sigma_{z1} = 0 \quad \text{or} \quad \partial^2\Phi/\partial Z^2 - \nabla^2\Phi = 0 \quad \text{for} \quad Z = 0,\ 0 \leqslant \rho < \infty \tag{2.4}$$

$$\sigma_{rz1} = 0 \quad \text{or} \quad \partial^2\Phi/\partial\rho\partial Z = 0 \quad \text{for} \quad Z = 0,\ 0 \leqslant \rho < \infty \tag{2.5}$$

$$u_r = 0 \quad \text{or} \quad \partial\Phi/\partial\rho = 0 \quad \text{for} \quad \rho = 0,\ 0 \leqslant Z < \infty \tag{2.6}$$

as well as the conditions

$$u_r = \partial\Phi/\partial\rho \to 0 \quad \text{as} \quad \rho \to \infty,\ u_z = \partial\Phi/\partial Z \to 0 \quad \text{as} \quad Z \to \infty \tag{2.7}$$

The solution of Eq. (2.2) that satisfies (2.4), (2.6) and (2.7), obtained by applying direct and inverse first-order Hankel and Laplace transformations in consecutive order has the form

$$\begin{aligned}
&\Phi(\rho, Z, t) = \Lambda\beta \int_0^\infty \varphi(\xi)\, \Phi_1(\xi, Z, \mathrm{Fo})\, J_0(\xi\rho)\, d\xi \\
&\Phi_1(\xi, Z, \mathrm{Fo}) = e^{-\xi Z}\left(\frac{\mathrm{Fo}}{2} - \frac{Z}{4\xi} - \frac{1}{4\xi^2}\right)\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} - \xi\sqrt{\mathrm{Fo}}\right) - \\
&- e^{\xi Z}\left(\frac{\mathrm{Fo}}{2} + \frac{Z}{4\xi} - \frac{1}{4\xi^2}\right)\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} + \xi\sqrt{\mathrm{Fo}}\right) - \\
&- e^{-\xi Z}\left(\mathrm{F}_0 - \frac{1}{2\xi^2}\right)\operatorname{erf}(\xi\sqrt{\mathrm{Fo}}) + e^{-\xi^2\mathrm{Fo}} \frac{\sqrt{\mathrm{Fo}}}{\xi\sqrt{\pi}}\left(e^{-Z^2/(4\mathrm{Fo})} - e^{-\xi Z}\right)
\end{aligned} \tag{2.8}$$

Using (2.8), we can find from (2.3) that

$$\begin{aligned}
&\sigma_{r1} = C\int_0^\infty \varphi(\xi)\, \Phi_1(\xi, Z, \mathrm{Fo})\left[\frac{\xi}{\rho} J_1(\xi\rho) - \xi^2 J_0(\xi\rho)\right] d\xi - C_1 T \\
&\sigma_{\theta 1} = -C\int_0^\infty \varphi(\xi)\, \Phi_1(\xi, Z, \mathrm{Fo})\, \frac{\xi}{\rho} J_1(\xi\rho)\, d\xi - C_1 T \\
&\sigma_{z1} = C\int_0^\infty \varphi(\xi)\, \Phi_1(\xi, Z, \mathrm{Fo})\, \xi^2 J_0(\xi\rho)\, d\xi \\
&\sigma_{rz1} = -C\int_0^\infty \varphi(\xi)\, \Phi_2(\xi, Z, \mathrm{Fo})\, \xi^2 J_1(\xi\rho)\, d\xi
\end{aligned} \tag{2.9}$$

Here

$$\begin{aligned}
&\Phi_2(\xi, Z, \mathrm{Fo}) = \left(\frac{Z}{4\xi} - \frac{\mathrm{Fo}}{2}\right)\left[e^{-\xi Z}\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} - \xi\sqrt{\mathrm{Fo}}\right) + \right. \\
&\left. + e^{\xi Z}\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} + \xi\sqrt{\mathrm{Fo}}\right)\right] + e^{-\xi Z}\left(\mathrm{Fo} - \frac{1}{2\xi^2}\right)\times \\
&\times \operatorname{erf}(\xi\sqrt{\mathrm{Fo}}) + \frac{\sqrt{\mathrm{Fo}}}{\xi\sqrt{\pi}} e^{-(\xi^2\mathrm{Fo} + \xi Z)}, \quad C = \Lambda C_1, \quad C_1 = 2\mu\beta a^{-2}
\end{aligned} \tag{2.10}$$

For $Z = 0$, taking (2.10) into account, we find from the last two relations in (2.9) that

$$\begin{aligned}
&\sigma_{rz1}(\rho, 0, t) = -C\int_0^\infty \varphi(\xi)\, \Phi_3(\xi, \mathrm{Fo})\, \xi^2 J_1(\xi\rho)\, d\xi \\
&\sigma_{z1}(\rho, 0, t) = 0 \\
&\Phi_3(\xi, \mathrm{Fo}) \equiv \Phi_2(\xi, 0, \mathrm{Fo}) = -\mathrm{Fo}\operatorname{erfc}(\xi\sqrt{\mathrm{Fo}}) - \\
&- \frac{1}{2\xi^2}\operatorname{erf}(\xi\sqrt{\mathrm{Fo}}) - e^{-\xi^2\mathrm{Fo}} \frac{\sqrt{\mathrm{Fo}}}{\xi\sqrt{\pi}}
\end{aligned} \tag{2.11}$$

It follows from (2.11) that boundary condition (2.5) is not satisfied. Therefore, we shall consider a supplementary problem concerned with distributed forces that act on the surface and induce stresses (denoted by the subscript 2) such that

$$\sigma_{z2} + \sigma_{z1} = 0, \quad \sigma_{rz2} + \sigma_{rz1} = 0 \quad \text{for} \quad Z = 0 \tag{2.12}$$

The stress state generated by forces whose distribution over the surface of the elastic half-space is axially symmetric can be determined using the Love function [4], which is the solution of the biharmonic equation

$$\nabla^4 L = 0 \tag{2.13}$$

The components of the stress tensor are connected with L by the relations

$$\sigma_{r2} = \frac{1}{a^3}\frac{\partial}{\partial Z}\left(\nu_1\nabla^2 L - \frac{\partial^2 L}{\partial \rho^2}\right), \quad \sigma_{\theta 2} = \frac{1}{a^3}\frac{\partial}{\partial Z}\left(\nu_1\nabla^2 L - \frac{1}{\rho}\frac{\partial L}{\partial \rho}\right)$$
$$\sigma_{z2} = \frac{1}{a^3}\frac{\partial}{\partial Z}\left[(2-\nu_1)\nabla^2 L - \frac{\partial^2 L}{\partial Z^2}\right], \quad \sigma_{rz2} = \frac{1}{a^3}\frac{\partial}{\partial \rho}\left[(1-\nu_1)\nabla^2 L - \frac{\partial^2 L}{\partial Z^2}\right] \tag{2.14}$$

The general solution of Eq. (2.13) bounded for $\rho \to \infty$ and $Z \to \infty$ has the form

$$L = \int_0^\infty e^{-\xi Z}(A_1 + ZA_2) J_0(\xi\rho)\, d\xi \tag{2.15}$$
$$\nabla^2 L = -\int_0^\infty 2\xi A_2 e^{-\xi Z} J_0(\xi\rho)\, d\xi$$

Substituting L from (2.15) into the boundary conditions (2.12) and taking the last two formulas in (2.14) into account, we get

$$A_1 = (2\nu_1 - 1)\xi^{-1}A_2, \quad A_2 = -Ca^3\varphi(\xi)\Phi_3$$

Then we can find from (2.15) that

$$L = -Ca^3 \int_0^\infty \varphi(\xi)\Phi_3(\xi, \mathrm{Fo})\left(\frac{2\nu_1 - 1}{\xi} + Z\right)e^{-\xi Z} J_0(\xi\rho)\, d\xi \tag{2.16}$$

We can obtain the total thermal stress field as the superposition of the stress fields (2.9) connected with the thermoelastic potential Φ and the stresses brought about by the Love function L, which are defined by (2.14). We have

$$\sigma_{rr} = C\int_0^\infty \varphi(\xi)\Big\{\Phi_1(\xi, Z, \mathrm{Fo})\left[\frac{\xi}{\rho}J_1(\xi\rho) - \xi^2 J_0(\xi\rho)\right] -$$
$$- \xi e^{-\xi Z}\Phi_3(\xi, \mathrm{Fo})\left[(2\xi - \xi^2 Z)J_0(\xi\rho) + (2\nu_1 - 2 + \xi Z)\frac{J_1(\xi\rho)}{\rho}\right]\Big\}d\xi - C_1 T$$

$$\sigma_{\theta\theta} = -C\int_0^\infty \varphi(\xi)\Big\{\Phi_1(\xi, Z, \mathrm{Fo})\frac{\xi}{\rho}J_1(\xi\rho) + \xi e^{-\xi Z}\Phi_3(\xi, \mathrm{Fo})\times$$
$$\times\left[2\nu_1\xi J_0(\xi\rho) - (2\nu_1 - 2 + \xi Z)\frac{J_1(\xi\rho)}{\rho}\right]\Big\}d\xi - C_1 T \tag{2.17}$$

$$\sigma_{zz} = C\int_0^\infty \varphi(\xi)\{\Phi_1(\xi, Z, \mathrm{Fo})\xi^2 J_0(\xi\rho) - \xi^3 Ze^{-\xi Z}\Phi_3(\xi, \mathrm{Fo})J_0(\xi\rho)\}\, d\xi$$

$$\sigma_{rz} = -C\int_0^\infty \varphi(\xi)\{\Phi_2(\xi, Z, \mathrm{Fo})\xi^2 J_1(\xi\rho) - \xi^2 e^{-\xi Z}\Phi_3(\xi, \mathrm{Fo})(1 - \xi Z)J_1(\xi\rho)\}\, d\xi$$

To evaluate the integrals in (2.17), we used the same numerical integration procedure [3] as that employed to find the temperature T given by (1.11).

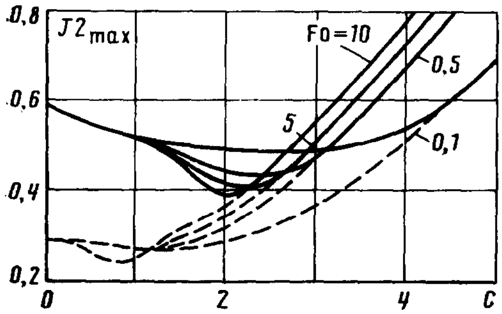

FIG. 2.

3. We represent the resulting stress field in the irregularity as the sum

$$\sigma_{ij} = p_0 [\bar{\sigma}_{ij}^i (X, \nu_1, f) + C\bar{\sigma}_{ij}^t (X, \nu_1, \mathrm{Fo})], \quad \bar{\sigma}_{ij}^i = \sigma_{ij}^i/p_0 \tag{3.1}$$

Here σ_{ij}^i are the isothermal stresses at an arbitrary point $X = (x, q, z)$ of the half-space due to the action of normal and shear forces on the boundary of the half-space inside the circular domain (the contact surface), $\bar{\sigma}_{ij}^t = \sigma_{ij}^t/(Cp_0)$, σ_{ij}^t are the thermal stresses (2.17), and, on the basis of (2.10), $C = E_1\alpha_1\gamma fVa/[\lambda(1-\nu_1)]$. Using (3.1), we can define the dimensionless Huber–Mises stress

$$J2 = \frac{1}{p_0}\left\{\frac{1}{6}[(\sigma_{rr}-\sigma_{\theta\theta})^2 + (\sigma_{\theta\theta}-\sigma_{zz})^2 + (\sigma_{zz}-\sigma_{rr})^2] + \sigma_{r\theta}^2 + \sigma_{\theta z}^2 + \sigma_{rz}^2\right\} \tag{3.2}$$

The case of an elastic half-space sliding on the surface of a tungsten ball has been investigated in detail. (Under high-temperature conditions, deposition of tungsten on a surface improves its friction properties [5].) The exact formulas for computing the isothermal stresses σ_{ij}^i are given in [6]. It is assumed that the stresses $\sigma_{zz} = -p(r)$ and $\sigma_{xz} = -fp(r)$ in the half-space caused by the normal and shear forces ($p(r)$ can be found from (1.2) with $m = 1$) are independent and the resulting stress–strain state can be found by taking their superposition. If the coefficient of friction is small (compared with unity), than this hypothesis is fully applicable [7].

Given the values of $f = 0.1$, C and Fo, we can use (3.2) to determine the stress $J2$ inside the rectangle $-1.5a \leq x \leq 1.5a$, $y = 0$, $0 \leq z \leq a$ with step $0.1a$ and inside the disc $r \leq 1.5a$, $z = 0$, $0° \leq \theta \leq 180°$ with step 15°.

The maximum values of $J2$ ($J2_{max}$) on the planes $y = 0$ and $z = 0$ are presented in Fig. 2 as functions of C for various values of Fo. As can be seen, there exists a local minimum of $J2_{max}$, and so, if the values are small, the thermal stresses arising in the irregularity due to the heat generated by friction reduce the total stress state level.

Let the strength σ_y of the material of the ball be a linearly decreasing function of the temperature,

$$\sigma_y = \sigma_y^\circ - nT \tag{3.3}$$

Here σ_y° is the isothermal strength of the material and n can be found experimentally or, for example, using the Ito–Shishokin formula [8]

$$H = H^* e^{-\alpha(273+T)} \tag{3.4}$$

for the hardness (H^* is the hardness at 20°C; in the case of tungsten $H^* = 4.35$ GPa). Since $\alpha = 0.002$ for metals, it follows that for small temperature gradients, we can find from (3.3) and (3.4) that $n = 1.68 \times 10^6$ Pa/degree. According to (1.1), the temperature inside the ball is equal to

$$T = p_0 C (1 - \nu_1)(E_1\alpha_t)^{-1} T^* \tag{3.5}$$

For tungsten, $E_1 = 344$ GPa, $\nu_1 = 0.3$, and $\alpha_t = 4.4 \times 10^{-6}$ degree^{-1}.

Taking (3.5) into account, we can write (3.3) in the form

$$\sigma_y/p_0 = \sigma_y^\circ/p_0 - 0{,}77CT^* \tag{3.6}$$

The relation

$$J2 \geq 3^{-1/2}\sigma_y/p_0$$

which serves as the criterion for Huber–Mises plastic flow and equality (3.6) yield

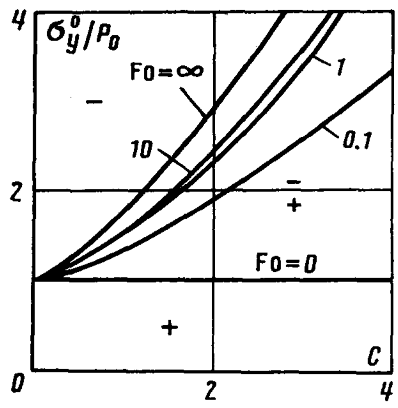

FIG. 3.

$$J2^t \geqslant \sigma_y/p_0,\quad J2^t = 3^{1/2}J2 + 0{,}77CT^* \tag{3.7}$$

The inequality (3.7) is the temperature-dependent plasticity criterion for tungsten. The material of the irregularity begins to flow at some point if the effective stress $J2^t$ at this point exceeds the relative tensile (compressive) strength.

On the surface $z = 0$ of the ball the inequality (3.7) can be written as

$$\max_{z=0} J2^t \geqslant \sigma_y^\circ/p_0 \tag{3.8}$$

The results of computing the left-hand side of (3.8) as a function of C for a number of values of the Fourier criterion and $f = 0.1$ are shown in Fig. 3. The horizontal straight line corresponds to the solution of the isothermal problem (Fo = 0). The relation between $J2$ and C is non-linear, since $J2^t$ depends on C through (3.1). In the case of materials for which plastic deformations play the predominant role in the process of wear of the contact surface, (3.8) serves as the criterion for the commencement of thermomechanical wear. The structure of C involves a constant part, namely, the mechanical and thermophysical properties of the material, as well as a variable part depending on the contact conditions γ, f and V. Every point in Fig. 3 is determined by σ_y°/p_0 and C. If the point lies above the curve Fo = 0, then there is no thermomechanical wear on the surface of the ball (the corresponding region is denoted by a minus sign). But if the point in question lies below the curve Fo = 0, then the wear of the surface begins as soon as the contact between the elastic bodies is established (the region is denoted by a plus sign). However, if the point lies between the curves Fo = 0 and Fo = ∞, the moment (Fo) when the wear (the flow) begins can be determined from Fig. 3. The domain of conditional wear is denoted by minus and plus signs. Thus, in terms of the given parameters σ_y°/p_0 and C, one can construct a picture of the plastic wear of the given material.

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